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ON SELECTION OF ZERO APPROXIMATION FOR THE ANGULAR POSITION OF AN ARTIFICIAL EARTH SATELLITE IN THE FORM OF TRIGONOMETRIC POLYNOMIALS TO THE FOURTH POWER

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16. Abstract

For high earth satellites, the spectral presentation for angles of orientation has comparatively large terms of the third and fourth power. Since in oriented motion the satellite, provided with a damper, is under conditions of forced oscillation, then even the approximating formulas for the angles of orientation can have large coefficients with harmonics of the third and fourth power. Research conducted shows that the use of approximating trigonometric polynomials of high powers also requires additional conditions which are independent of the magnetometer readings.

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ON SELECTION OF A ZERO APPROXIMATION FOR THE ANGULAR POSITION OF AN ARTIFICAL EARTH SATELLITE IN THE FORM OF TRIGONOMETRIC POLYNOMIALS TO THE FOURTH POWER

V. S. Novoselov

1. Statement of the Problem /173

In the work of author Novoselov [1] is suggested a method for selecting the zero approximation in defining the angular position of an artifical Earth satellite [AES] in oriented motion in the form of trigonometric polynomials in powers of the argument of latitude. For low AES, the spectral presentation for perturbing moments does not contain terms of more than the second power [2]. Thus, in this study, approximating formulas for angles of orientation were also used in the form of trigonometric polynomials to the second power. But for high AES, extremely substantial value is acquired by the perturbing moment of light pressure. The spectral presentation for such moment has a comparatively large terms of the third and fourth power. Since in oriented motion the satellite, provided with a damper, is in conditions of forced oscillation, then even the approximating formulas for the angles of orientation can have large coefficients with harmonics of the third and fourth power. Below is conducted a discussion of the question of selecting trigonometric polynomials to the fourth power approximating angles of AES orientation on the basis of magnetometer data.

We will discuss oriented motion of an AES in a circular orbit and its angular position will be defined using 'aircraft' angles: yaw ψ, bank ϕ, and pitch θ. Let us employ the following approximation:

$$
\left.
\begin{aligned}
\psi &= D_{11} + \sum_{n=1}^{4} (D_{1,2n} \sin nu + D_{1,2n+1} \cos nu), \\
\varphi &= D_{21} + \sum_{n=1}^{4} (D_{2,2n} \sin nu + D_{2,2n+1} \cos nu), \\
\theta &= D_{31} + \sum_{n=1}^{4} (D_{3,2n} \sin nu + D_{3,2n+1} \cos nu).
\end{aligned}
\right\} \quad (1)
$$

In formula (1) D_{p1}, $D_{p,\,2n}$, $D_{p,\,2n+1}$ ($p = 1, 2, 3$; $n = 1, 2, 3, 4$) are constants subject to definition; u--the argument of latitude. For the circular orbit in equation, $u = \omega_0 t$, where ω_0--orbital angular velocity, t--time measured from the moment of passage through the equator.

As in study [1], let us assume that on the satellite is set a magnetometer measuring the projections H_x, H_y, H_z of geomagnetic field strength $\bar{H}$ with respect to the constructive system of coordinates (x, y, z). Starting with a dipole model of the Earth's magnetic field and employing formulas (1) and (2) of study [1], we find that

$$
\begin{aligned}
H_0^{-1} H_x = {} & D_{11} \cos i + D_{32} \sin i + \sin u \, [D_{12} \cos i + (2D_{31} - D_{35}) \sin i] + \\
& + \cos u \, [D_{13} \cos i + (1 + D_{34}) \sin i] + \sin 2u \, [D_{14} \cos i + (D_{33} - D_{37}) \sin i] + \\
& + \cos 2u \, [D_{15} \cos i + (D_{36} - D_{32}) \sin i] + \sin 3u \, [D_{16} \cos i + (D_{35} - D_{39}) \sin i] + \\
& + \cos 3u \, [D_{17} \cos i + (D_{38} - D_{34}) \sin i] + \sin 4u \, (D_{18} \cos i + D_{37} \sin i) + \\
& + \cos 4u \, (D_{19} \cos i - D_{36} \sin i) + D_{39} \sin i \sin 5u - D_{38} \sin i \cos 5u,
\end{aligned}
\quad (2)
$$

$$
\begin{aligned}
H_0^{-1} H_y = {} & \cos i - \left(\frac{1}{2} D_{13} + D_{22}\right) \sin i + \sin u \sin i \left(-\frac{1}{2} D_{14} - 2D_{21} + D_{25}\right) + \\
& + \cos u \sin i \left(-D_{11} - \frac{1}{2} D_{15} - D_{24}\right) + \sin 2u \sin i \left[-\frac{1}{2} (D_{12} + D_{16}) - \right. \\
& \left. - (D_{23} - D_{27})\right] + \cos 2u \sin i \left[-\frac{1}{2} (D_{13} + D_{17}) - (D_{26} - D_{22})\right] + \\
& + \sin 3u \sin i \left[-\frac{1}{2} (D_{14} + D_{18}) - (D_{25} - D_{29})\right] + \\
& + \cos 3u \sin i \left[-\frac{1}{2} (D_{15} + D_{19}) + (D_{24} - D_{28})\right] + \\
& + \sin 4u \sin i \left(-\frac{1}{2} D_{16} - D_{27}\right) + \cos 4u \sin i \left(-\frac{1}{2} D_{17} + D_{28}\right) + \\
& + \sin 5u \sin i \left(-\frac{1}{2} D_{18} - D_{29}\right) + \cos 5u \sin i \left(-\frac{1}{2} D_{19} - D_{28}\right),
\end{aligned}
\quad (3)
$$

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$$
\begin{aligned}
H_0^{-1}H_z = {} & -D_{21}\cos i + \frac{1}{2} D_{33} \sin i + \sin u \left[-D_{22} \cos i + \left(-2 + \frac{1}{2} D_{34} \right) \sin i \right] + \\
& + \cos u \left[-D_{23} \cos i + \left(D_{31} + \frac{1}{2} D_{35} \right) \sin i \right] + \\
& + \sin 2u \left[-D_{24} \cos i + \frac{1}{2} (D_{32} + D_{36}) \sin i \right] + \\
& + \cos 2u \left[-D_{25} \cos i + \frac{1}{2} (D_{33} + D_{37}) \sin i \right] + \\
& + \sin 3u \left[-D_{30} \cos i + \frac{1}{2} (D_{34} + D_{38}) \sin i \right] + \\
& + \cos 3u \left[-D_{27} \cos i + \frac{1}{2} (D_{35} + D_{39}) \sin i \right] + \\
& + \sin 4u \left(-D_{28} \cos i + \frac{1}{2} D_{36} \sin i \right) + \cos 4u \left(-D_{29} \cos i + \frac{1}{2} D_{37} \sin i \right) + \\
& + \frac{1}{2} \sin 5u \sin i \, D_{38} + \frac{1}{2} \cos 5u \sin i \, D_{39}.
\end{aligned}
\tag{4}
$$

Here i denotes the orbital slope of the AES and the equatorial plane; H_0 will denote some constant typical for the altitude of flight of the satellite.

Formulas (2)-(4) show that it is expedient to approximate the table of readings of the magnetometer in the form of trigonometric polynomials by powers of the argument of latitude u with specification of terms to the fifth power exclusively:

$$
H_x = H_{11} + \sum_{n=1}^{5} H_{1,2n} \sin nu + H_{1,2n+1} \cos nu, \tag{5}
$$

$$
H_y = H_{21} + \sum_{n=1}^{5} H_{2,2n} \sin nu + H_{2,2n+1} \cos nu, \tag{6}
$$

$$
H_z = H_{31} + \sum_{n=1}^{5} H_{3,2n} \sin nu + H_{3,2n+1} \cos nu. \tag{7}
$$

Let us discuss the problems formulated in study [1] as they apply to the use of formulas (2)-(7).

2. Insufficiency of magnetometer readings for selection of a zero approximation of the angular position of an oriented artificial Earth satellite /175

Let us equate free terms and also the coefficients for identical trigonometric functions in formulas (2)-(4) and (5)-(7) previously multiplied by H_0. Since the problem of selecting a zero approximation for D_{p1}, $D_{p,\ 2n}$, $D_{p,\ 2n+1}$ is being solved, these coefficients will be supplied with a zero index. Equations derived from the equation of free terms and coefficients where sin u and cos u exist, have the form of the first three equations for each of the systems of notations (12)-(14) of study [1]. The remaining equations are written as follows:

$$\left.\begin{array}{l}
D^0_{14}\cos i+(D^0_{33}-D^0_{37})\sin i=H_{14}H_0^{-1},\\
D^0_{15}\cos i+(-D^0_{32}+D^0_{36})\sin i=H_{15}H_0^{-1},\\
D^0_{16}\cos i+(D^0_{35}-D^0_{39})\sin i=H_{16}H_0^{-1},\\
D^0_{17}\cos i+(-D^0_{34}+D^0_{38})\sin i=H_{17}H_0^{-1},\\
D^0_{18}\cos i+D^0_{37}\sin i=H_{18}H_0^{-1},\\
D^0_{19}\cos i-D^0_{36}\sin i=H_{19}H_0^{-1},\\
D^0_{39}\sin i=H_{1,10}H_0^{-1},\qquad -D^0_{38}\sin i=H_{1,11}H_0^{-1};
\end{array}\right\} \tag{8}$$

$$\left.\begin{array}{l}
-\frac{1}{2}D^0_{12}-\frac{1}{2}D^0_{16}-D^0_{23}+D^0_{27}=H_{24}(H_0\sin i)^{-1},\\
-\frac{1}{2}D^0_{13}-\frac{1}{2}D^0_{17}+D^0_{22}-D^0_{26}=H_{25}(H_0\sin i)^{-1},\\
-\frac{1}{2}D^0_{14}-\frac{1}{2}D^0_{18}-D^0_{25}+D^0_{29}=H_{26}(H_0\sin i)^{-1},\\
-\frac{1}{2}D^0_{15}-\frac{1}{2}D^0_{19}+D^0_{21}-D^0_{28}=H_{27}(H_0\sin i)^{-1},\\
-\frac{1}{2}D^0_{16}-D^0_{27}=H_{28}(H_0\sin i)^{-1},\\
-\frac{1}{2}D^0_{17}+D^0_{26}=H_{29}(H_0\sin i)^{-1},\\
-\frac{1}{2}D^0_{18}-D^0_{29}=H_{2,10}(H_0\sin i)^{-1},\\
-\frac{1}{2}D^0_{19}+D^0_{28}=H_{2,11}(H_0\sin i)^{-1};
\end{array}\right\} \tag{9}$$

$$\left.\begin{array}{l}
-D^0_{24}\cos i+\frac{1}{2}(D^0_{32}+D^0_{36})\sin i=H_{34}H_0^{-1},\\
-D^0_{25}\cos i+\frac{1}{2}(D^0_{33}+D^0_{37})\sin i=H_{35}H_0^{-1},\\
-D^0_{26}\cos i+\frac{1}{2}(D^0_{34}+D^0_{38})\sin i=H_{36}H_0^{-1},\\
-D^0_{27}\cos i+\frac{1}{2}(D^0_{35}+D^0_{39})\sin i=H_{37}H_0^{-1},\\
-D^0_{28}\cos i+\frac{1}{2}D^0_{36}\sin i=H_{38}H_0^{-1},\\
-D^0_{29}\cos i+\frac{1}{2}D^0_{37}\sin i=H_{39}H_0^{-1},\\
\frac{1}{2}D^0_{38}\sin i=H_{3,10}H_0^{-1},\qquad \frac{1}{2}D^0_{39}\sin i=H_{3,11}H_0^{-1}.
\end{array}\right\} \tag{10}$$

We have a system of 33 equations to define 27 unknowns. But these /176 same equations, which are in study [1], are dependent on the remaining equations. Given the following numeration of equations: from 1 through 11--the first three equations of system (12) of study [1] and equation (8); from 12 to 22--the first three equations of system (13) of study [1] and equation (9); from 23 to 33--the first three equations of system (14) of study [1] and equation (10). We will examine the non-degenerate case $i \neq \{0, 90°, 180\}$. For the numeration adopted, the dependents are equations No: 3, 5, 13, 14, 25, 26, 27. In reality, by substituting instead of $H_{p1}H_0^{-1}$, $H_{p,2n}H_0^{-1}$, $H_{p,2n+1}H_0^{-1}$ the corresponding left sides of the equations, we will have the identities

$$\left.\begin{aligned}
&(H_{13}+2\operatorname{ctg} i\, H_{21}-2H_{32})\, H_0^{-1}-(2\sin^{-1} i+3\sin i)\equiv 0,\\
&(H_{15}+H_{19}+2\operatorname{ctg} i\, H_{27}+2H_{34}-2H_{38})\, H_0^{-1}\equiv 0,\\
&\left(H_{22}+\frac{1}{2}\operatorname{tg} i\, H_{14}-2\operatorname{tg} i\, H_{31}+\operatorname{tg} i\, H_{35}\right) H_0^{-1}\equiv 0,\\
&\left(H_{23}+\operatorname{tg} i\, H_{11}+\frac{1}{2}\operatorname{tg} i\, H_{15}-\operatorname{tg} i\, H_{34}\right) H_0^{-1}\equiv 0,\\
&\left(H_{33}-\frac{1}{2}H_{12}-\frac{1}{2}H_{16}-\operatorname{ctg} i\, H_{24}-H_{37}\right) H_0^{-1}\equiv 0,\\
&\left(H_{34}+\frac{1}{2}H_{15}+\frac{1}{2}H_{19}+\operatorname{ctg} i\, H_{27}-H_{38}\right) H_0^{-1}\equiv 0,\\
&\left(H_{35}-\frac{1}{2}H_{14}-\frac{1}{2}H_{18}-\operatorname{ctg} i\, H_{26}-H_{39}\right) H_0^{-1}\equiv 0.
\end{aligned}\right\} \tag{11}$$

Moreover, the last two equations of system (8) and system (10) are equivalent. From the equations we find that

$$\left.\begin{aligned}
&D_{38}^0=-H_{1,11}(H_0\sin i)^{-1}=2H_{3,10}(H_0\sin i)^{-1},\\
&D_{39}^0=H_{1,10}(H_0\sin i)^{-1}=2H_{3,11}(H_0\sin i)^{-1}.
\end{aligned}\right\} \tag{12}$$

Thus, from a number of unknowns are eliminated D_{38}^0 and D_{39}^0; and from the total number of equations we must exclude, on the basis of identities in (11), seven equations of those indicated above; also on the basis of equations in (12), four equations Nos. 10, 11, 32, 33. We have a system of 22 equations to define 25 unknowns.

As in study [1], by using the data only of the magnetometer, we will be short at least three independent equations. An increase in the order of the approximating trigonometric polynomials did not bring about a change in the total position, due to the lack of data of only the magnetometer to define the angular position of the oriented AES. Approach (A) according to the terminology of study [1] was used above.

Given we are using the simplified approach of type (B) of study [1], with truncated approximating formulas for the readings of the magnetometer. In this case, approach (B) employs a summation with respect to n in formulas (5)-(7) from 1 to 4. In approach (B) the last two equations in each of systems (8)-(10) will be dropped. We yield with approach (B) a system of 20 equations to define 27 unknowns.

Let us now discuss degenerate cases. Given that sin i = 0. By studying the above system of 33 equations in approaches (A) and (B) we find that

$$\begin{gathered} D_{11}^{0}=H_{11}(H_0\cos i)^{-1},\quad D_{1,2n}^{0}=H_{1,2n}(H_0\cos i)^{-1},\quad D_{1,2n+1}=H_{1,2n+1}(H_0\cos i)^{-1},\\ D_{21}^{0}=-H_{31}(H_0\cos i)^{-1},\quad D_{2,2n}^{0}=H_{3,2n}(H_0\cos i)^{-1},\quad D_{2,2n+1}=H_{3,2n+1}(H_0\cos i)^{-1},\\ n=1,\ 2,\ 3,\ 4,\ 5. \end{gathered} \tag{13}$$

The coefficients D_{31}^{0}; $D_{3,2n}^{0}$ and $D_{3,2n+1}^{0}$ are not defined. As in study [1], we come to the conclusion that in motion along an equatorial orbit, we can unambiguously define oscillation in angles ψ and ϕ using a magnetometer and be unable to define pitching oscillations of the satellite in that orbit. /177

For the second degenerate case i = 90° and the coefficients of the approximating polynomial of angle θ are defined by approach

(A) unambiguously both by readings of H_x and by readings of H_y in the form

$$
\begin{gathered}
D_{31}^{0}=\frac{1}{2}\left(H_{12}+H_{16}+H_{1,10}\right) H_0^{-1}=\left[H_{33}-2\left(H_{37}-H_{3,11}\right)\right] H_0^{-1}= \\
=\frac{1}{2}\left(H_{13}+\frac{1}{2} H_{16}+H_{37}\right) H_0^{-1}, \quad D_{32}^{0}=H_{11} H_0^{-1}=2\left(H_{34}-H_{38}\right) H_0^{-1}, \\
D_{33}^{0}=\left(H_{14}+H_{18}\right) H_0^{-1}=2 H_{31} H_0^{-1}, \quad D_{34}^{0}=H_{13} H_0^{-1}=2 H_{31} H_0^{-1}, \\
D_{35}^{0}=\left(H_{16}+H_{1,10}\right) H_0^{-1}=2\left(H_{37}-H_{3,11}\right) H_0^{-1}=\left(H_{37}+\frac{1}{2} H_{18}\right) H_0^{-1}, \\
D_{36}^{0}=-H_{19} H_0^{-1}=2 H_{38} H_0^{-1}, \quad D_{37}^{0}=H_{18} H_0^{-1}=2 H_{39} H_0^{-1}, \\
D_{38}^{0}=H_{1,10} H_0^{-1}=2 H_{3,10} H_0^{-1}=\left(H_{17}+H_{13}\right) H_0^{-1}=2\left(H_{36}-H_{32}\right) H_0^{-1}, \\
D_{39}^{0}=-H_{1,11} H_0^{-1}=2 H_{3,11} H_0^{-1}=\left(H_{18}-H_{16}\right) H_0^{-1} .
\end{gathered} \tag{14}
$$

But to define the 18 coefficients of the approximating polynomials for angles ψ and ϕ we have only 11 equations. Since according to formulas (14) we can calculate the coefficients without knowing $H_{1,10}$; $H_{1,11}$; $H_{3,10}$; $H_{3,11}$, then the coefficients for angle θ are uniquely defined with approach (B). With approach (B) for the remaining 18 coefficients we will have only 7 equations.

The research conducted shows that the use of approximating trigonometric polynomials of high powers also requires additional conditions which are independent of the magnetometer readings. As in study [1], we can construct an algorithm of selection of the zero approximation of angles of orientation in the presence of additional information on the value of the angles of orientation at several points in the orbit of motion of the satellite--or on the direction of the second vector also at several points of the orbit of this satellite.

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